二维变厚板的 J 积分分析

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一、二维变厚板的 J 积分扩充表达式

本文将 J. R. Rice 提出的 J 积分公式扩充应用到二维变厚板。本文把变厚度的影响以修正项形式引入 J 积分公式，从而推导出二维变厚板的 J 积分扩充表达式。

首先研究无体积力条件下变厚板的平衡方程式，对图 1 所示的微元体，在 x_1 和 x_2 方向上，根据平衡条件导出各应力分量之间的关系式。

$$\left.\begin{aligned} &\partial\sigma_{11}/\partial x_1 + \partial\sigma_{12}/\partial x_2 + 1/B(\sigma_{11}\partial B/\partial x_1 + \sigma_{12}\partial B/\partial x_2) = 0 \\ &\partial\sigma_{21}/\partial x_1 + \partial\sigma_{22}/\partial x_2 + 1/B(\sigma_{21}\partial B/\partial x_1 + \sigma_{22}\partial B/\partial x_2) = 0 \end{aligned}\right\} \tag{1}$$

或

$$\sigma_{ij,j} + (1/B)\sigma_{ik}B_{,k} = 0 \quad (i, j, k = 1, 2) \tag{2}$$

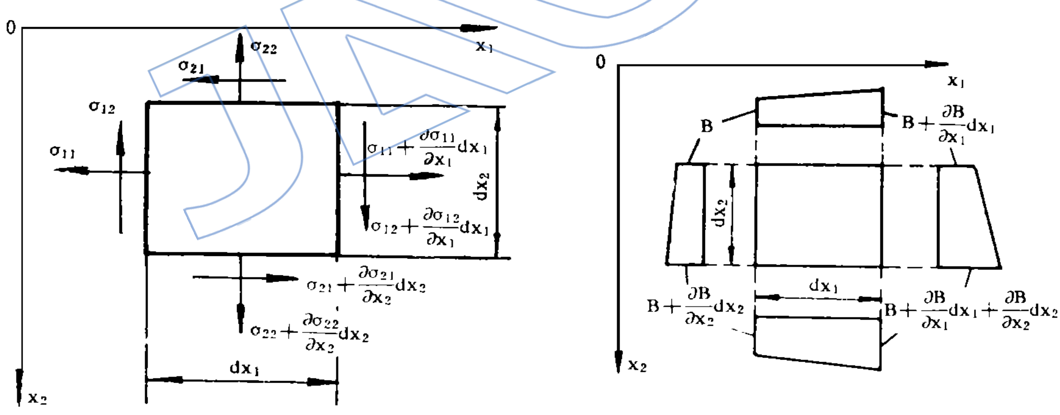

图 1　二维变厚板的力平衡图

式中的 B 表示板的厚度。然后推导 J 积分的扩充表达式。在小应变和单调加载条件下，如按图 2 取积分回路，即 $\Gamma^* = \Gamma_1 + S_1 - F_2 + S_2$。

设

$$I = \int_{\Gamma^*}\left(w\,dx_2 - p_i\frac{\partial u_i}{\partial x_1}ds\right) \quad (i, j = 1, 2) \tag{3}$$

式中 w 为应变能密度，p_i 和 u_i 分别为积分回路微段 ds 上的应力矢量和位移矢量。可证明[1]

$$I = \int_{A^*}\frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA \quad (i, k = 1, 2) \tag{4}$$

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显然可见，(4) 式右端不等于零。如设

$$J = I - \int_{A^*} \frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA \quad (i, k = 1, 2) \quad (5)$$

则 $J = \int_{\Gamma^*}\left[w dx_2 - p_i \frac{\partial u_i}{\partial x_1}ds\right] - \int_{A^*}\frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA = 0$

当裂纹表面无外力作用，且 $A^* = A_1 - A_2$，则可证明 $J_1 = J_2$。其中

$$J_1 = \int_{\Gamma_1}\left[w dx_1 - p_i \frac{\partial u_i}{\partial x_1}ds\right] - \int_{A_1}\frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA$$

$$J_2 = \int_{\Gamma_2}\left[w dx_1 - p_i \frac{\partial u_i}{\partial x_1}ds\right] - \int_{A_2}\frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA$$

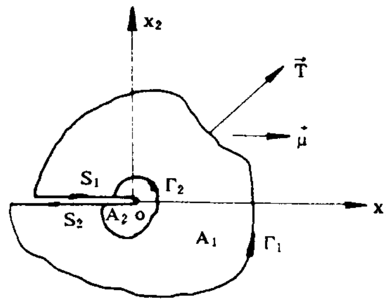

图 2 J 积分回路示意图

因 Γ_1 和 Γ_2 均为任意积分回路故二维变厚板的 J 积分扩充表达式具有守恒性，即

$$J = \int_{\Gamma}\left[w dx_2 - p_i \frac{\partial u_i}{\partial x_1}ds\right] - \int_{A}\frac{1}{B}\sigma_{ik}B_{,k}\frac{\partial u_i}{\partial x_1}dA \qquad (6)$$

二、J 积分扩充表达式的有限元数值计算

将 (6) 式中的积分号用求和式代替，微分号用有限差代替，则在常规的二维有限元程序[2]加 J 积分子程序后可算得J 积分值。(6) 式中右端第一项线积分为 *Rice* 公式，以 J_R 表示；第二项面积分为变厚度影响的修正项，以 $-J_B$ 表示。则可将 (6) 式改写为

$$J = J_R + J_B \qquad (7)$$

为了考核 (6) 式的守恒性，计算了单边和双边裂纹的变厚板模型，其厚度变化率分别取 0.120、0.207、0.204 和 1.00。计算时取三条积分回路，计算结果见表 1。

表 1 两种计算模型的 J 积分计算表

模型		单边裂纹变厚板		双边裂纹变厚板	
J 积分	回路	0.120	0.207	0.204	1.00
J_R	1	0.105×10^{-1}	0.207×10^{-1}	0.391×10^{-1}	0.118×10^{-1}
	2	0.108×10^{-1}	0.224×10^{-1}	0.409×10^{-1}	0.126×10^{-1}
	3	0.111×10^{-1}	0.234×10^{-1}	0.419×10^{-1}	0.129×10^{-1}
J_B	1	-0.607×10^{-3}	-0.263×10^{-2}	-0.271×10^{-2}	0.151×10^{-2}
	2	-0.873×10^{-3}	-0.399×10^{-2}	-0.378×10^{-2}	0.211×10^{-2}
	3	-0.107×10^{-2}	-0.497×10^{-2}	-0.445×10^{-2}	0.219×10^{-2}
J	1	0.9893×10^{-2}	0.1807×10^{-1}	0.3639×10^{-1}	0.1331×10^{-1}
	2	0.9927×10^{-2}	0.1841×10^{-1}	0.3712×10^{-1}	0.1471×10^{-1}
	3	1.003×10^{-2}	0.1843×10^{-1}	0.3745×10^{-1}	0.1509×10^{-1}
守恒性误差 Δ%		1.4	1.8	2.8	11.8

表 1 中的守恒性误差 Δ 为三条积分回路中 J 的最大值减去最小值，再除以最大值。表 1 的计算结果表明，对于二维变厚板，*Rice* 的 J 积分 J_R 是不守恒的，但加上修正项 J_B 后才基

本守恒。在同样分割网格条件下，厚度变化率愈大，守恒性误差愈大。此时应进一步细化网格，才能提高计算精度，减少守恒性误差。

三、J 积分扩充表达式的数值计算与光弹试验结果对比

为了进一步验证二维变厚板 J 积分扩充表达式的正确性，本文对环氧树脂制成的单边和双边裂纹变厚板试件进行了数值计算和光弹试验。在光弹试验中，为了较精确地求出应力强度因子值，采用了三向径法[3]和多点法相结合的综合法处理试验数据。先用三向径法进行粗略计算，将其结果作为多点法的初始值，再作详细计算，这可使计算时间缩短而将精度提高。光弹试验结果和用三条积分回路的 J 积分扩充表达式的计算结果均列在表 2 中。

表 2 J 积分扩充表达式的计算结果与光弹试验结果对比表

变厚板试件	光弹试验值	J 积分扩充表达式计算值			误 差
类 型	K_I (kgf/mm$^{3/2}$)	J (kgf/mm)	K_I (kgf/mm$^{3/2}$)	$K_{I平均}$	(%)
		0.01804	2.6711		
单边裂纹	2.6298	0.01839	2.6969	2.6975	2.49
		0.01869	2.7191		
		0.02405	3.0845		
双边裂纹	2.9931	0.02416	3.0915	3.0914	3.20
		0.02429	3.0998		

结论 综上所述，可以得到以下三点结论：(1) 二维变厚板的 J 积分扩充表达式在小变形、单调加载和无体积力条件下，具有守恒性。因此，可以用作裂纹尖端的断裂参量。(2) 在工程上可以用常规的二维有限元程序算出变厚板的 J 积分值，该值与光弹性试验值比较，误差在 4% 以内。(3) 对于厚度变化较大的板件，必须进一步细化计算网格，或用三维 J 积分公式计算，才能保证有一定的计算精度，使守恒性误差足够小。

参 考 文 献

[1] 聂景旭、洪其麟、郑光华、聂聪，“断裂力学理论及其在发动机上的应用”，航空专业教材编审组出版，1985。

[2] E. Hinton, D. R. J. Owen, “Finite Element Programming”, 1977.

[3] 顾求林等，“二维光弹性测定应力强度因子 K_I 的双向经法和三向径法”，清华大学论文，1981。

the references for searching the eigenvalues are the first k-th powers of the eigen determinants (where k is larger than 1.0) instead of the determinants themselves, and the searching formulas of the existing method are modified correspondingly. As the range of the references for searching the eigenvalues is reduced considerably, the phenomenon of numerical overflow is eliminated forever.

J- INTEGRAL FOR 2-D VARIABLE- THICKNESS PLATE

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ABSTRACT The formula of a J-integral presented by J. R. Rice is extended to problem of 2-D variable-thickness plates. The extended J-integral formula is derived in terms of the path independent. Then the integral conservation is verified by using finite element method. At last, the calculation results for some variable thickness plates made of photoelastic material are compared with their photoelastic test data, and the differences between them are less than 4%. It is proved that the extended J-integral formula for the 2-D variable-thickness plates is path independent and can be taken as a fracture paramarater for damage tolerance analysis. Furtheremore, this integral can be solved numerically by using 2-D general finite element programs. It is therefore of practical value for the fracture analysis of Ⅰ mode cracked plate with thickness which does not vary too mueh.

COMBUSTION PROCESSES OF HYDROGEN IN A SUPERSONIC AIR FLOW

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ABSTRACT According to the implicit numerical integration method proposed by Tyson. J, an improved computer program for pseudo-one-dimensional flow of combustion kinetics is developed with the aid of a chemical kinectic model with 12-species and 31 reaction. Variations of temperature, prssure, Mach number and mass fraction of chemical species in a supersonic combustion model are studied in detail. The results of scramjet combustor conditions corresponding to flight Mach numbers 6, 10 and 15 show that chemical reaction rates decrease at rather low temperature and pressure at a combustor inlet and there is not enough time to ignite the mixture-gas in the combustor at very high Mach number, in result the performance limits of scramjet kinetic combustion occur.

CHARACTERISTICS OF DUAL SWIRLERS SPRAYER DEVICE

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ABSTRACT The spray characteristics of dual radial swirlers mixing-cup sprayer device have been investigated with the aid of Malvern laser granulometer and checked the differences of its spray characteristics from those of the pressure atomizing fuel nozzle. Two air flow proportions of the first swirler to the second one are 50%∶50% and 30%∶70%. The airflow velocity at the exit-section has been designed in the range 37 to 74m/s. In the tests the fuel pressure ranges from 2.94 to 4.41 × 10^5Pa. The test results show that in the case of reasonable design, the sprayer device is able to provide better spray characteristics than pressure atomizing fuel nozzle.